XXVII. *An abstract of the results deduced from the measurement of an arc on the meridian, extending from latitude 8° 9′ 38″,4, to latitude 18° 3′ 23″,6, N. being an amplitude of 9° 53′ 45″,2. By Lieut. Colonel* William Lambton, *F. R. S. 33d Regiment of foot.*

Read May 21, 1818.

In the 12th vol. of the Asiatick Researches, there are detailed accounts of two complete sections of an arc on the meridian, measured by me at different times, in prosecuting the trigonometrical survey of the Peninsula of India. The first is comprehended between the parallels of Punnæ, a station near Cape Comorin, in latitude 8° 9′ 38″,39, and Patchipolliam in Coimbetoor, in latitude 10° 59′ 48″,93; being an amplitude of 2° 50′ 10″,54. The second is comprehended between the parallels of Patchipolliam and Namthabad, a station near Gooty, in the ceded districts; and lying in latitude 15° 6′ 0″,21, gives an amplitude of 4° 6′ 11″,28. Since these measurements were made, I have had the good fortune to get another section, extending from Namthabad to Daumergidda, in the Nizam's dominions, which being in latitude 18° 3′ 23″,6, gives an increase of 2° 57′ 23″,32; making in the whole an arc of 9° 53′ 45″,14 in amplitude; the longest *single* arc that has ever been measured on the surface of this globe. The detailed accounts of this last arc, with various conclusions concerning the three sections, have been presented to the Asiatick Society by the Marquess of Hastings, the present Governor General, and will be published in the 13th volume of the Asiatick Researches. But as it may be three or four years before that volume makes its appearance, I have been induced to draw out this abstract of the results, thinking that the conclusions herein contained may be interesting to the Royal Society, and to the astronomers of Europe. As a reference may be made to the volumes above mentioned, I have simply given the lengths of the sides of the triangles from which the arcs are deduced, together with the lengths of the terrestrial and celestial arcs, without including either the tables of triangles, or the particulars of the astronomical observations, farther than the names of the stars, and the mean lengths of the celestial arcs, and, consequently, the mean degrees deduced therefrom.

The first of these sections gives the degree due to latitude 9° 34′ 44″, the middle point of that arc, equal 60472,83 fathoms. The second section, whose middle point is in latitude 13° 2′ 55″, gives the mean degree equal 60487,56 fathoms. The last section gives the degree equal 60512,78 fathoms, due to the latitude of 16° 34′ 42″, the middle point of that section.

In my second paper, in the 12th vol. of the Asiatick Researches, it appeared that the degree due to latitude 11° 37′ 49″, the middle point between Punnæ and Namthabad, was 60480,3 fathoms. Since that paper was sent, there has been a small correction applied to the base near Gooty, after comparing the chains with the brass standard scale. This correction has somewhat increased the meridional distance between that base and Yerracondah south; and, consequently, the whole terrestrial arc between Namthabad and Punnæ is also increased, which now gives the degree due to latitude

11° 37′ 49″, equal 60481,55 fathoms. However, as there are now three distinct sections, whose respective middle points lie in 9° 34′ 44″; 13° 2′ 55″; and 16° 34′ 42″; I have thought it best to take the degrees due to these latitudes, as deduced from actual observation, using each, *first* with the French measure, *then* with the English, and *lastly* with the Swedish measure; and thence obtaining a general mean for the compression at the poles. The *first mean* of these three degrees used with the French degree, gives the compression $\frac{1}{309,15}$. The *second mean* of the same three degrees used with the English degree, gives $\frac{1}{313,54}$. And the *third* mean of these three degrees used with the Swedish degree, gives $\frac{1}{307,19}$ for the compression; so that the mean of these three means will give the compression at the poles $\frac{1}{309,96}$, or $\frac{1}{310}$ nearly of the polar axes; and this has been finally adopted for computing the general tables of degrees from the equator to the pole.

It will be seen by inspecting the plan of the triangles, (Pl. XXIII.) that all the sides from which the arc has been deduced lie so near the meridian, that no correction has been required; a circumstance that has saved much trouble. The sides being so nearly north and south, that the base reduced from each side as an hypothenuse, may be considered as a chord of an arc parallel, and so nearly contiguous to the meridian, that it may, as to sense, be taken as the chord to the same arc on the meridian; and these chords being in general short, they will be the same as the arcs, very nearly. I have therefore not been at the trouble of applying any correction; for if the whole arc between Punnæ and Daumergidda (upwards of 680 miles) were divided into small arcs of 30 miles each, the whole difference between these arcs and their chords would not be more than a fathom and a half.

The number of base lines in this extensive arc, are five; all measured with the chain extended in coffers; with elevating screws, &c; and every part of the operation has been performed with the greatest posssible attention. The one near Bangalore may be considered as the first; and its height above the sea was obtained by a series of triangles connecting it with another base near St. Thomas's Mount, whose height above the low water mark was determined by observations made at the sea beach, and at the race stand near the north end of the base (Asiat. Res. vol.viii.). The base lines to the southward are, the one in Coimbetoor, and the other near Tinnivelly, whose heights above the sea were determined from the Bangalore base. Those to the northward are, the base near Gooty, and the one near Daumergidda. The account of this last measurement, and of the curious experiments for comparing the steel chains with the brass standard scale, will appear in the 13th volume of the Asiatick Researches. The particulars of the other measurements may be seen in the 10th and 12th volumes of the same work.

The great station of observation at Doddaguntah, is near the first base line; and it was at that station where the position of the meridian was fixed for extending it to the north and south. The latitude of that station was also determined by observing the zenith distances of a number of stars from the Greenwich catalogue for 1802. That latitude, however, was afterwards set aside from a supposed disturbance of the plummet. The latitude was afterwards fixed from observations made at Punnæ, the southernmost station of observation, by setting off degrees corresponding to different latitudes, after they were finally determined.

As Doddaguntah station has been left out in the divisions of the grand arc, its latitude was only useful in fixing that of Savendroog, one of the great meridian stations for crossing the Peninsula (Asiatick Researches, vol. i.). The other remaining stations of observation are, Patchipolliam, or Coimbetoor; Namthabad, near Gooty; and Daumergidda, in the Nizam's dominions. We shall now see how these stations are connected by the triangles; Doddaguntah being the referring station in counting the northings, southings, &c.; and to whose meridian the whole terrestrial arc is reduced.

Without regarding the order of time, we will set off from Doddaguntah to Patchipolliam, and thence to Punnæ. After which we will return to Doddaguntah, and proceed northerly to Namthabad, and from Namthabad to Daumergidda.

TABLE I. *Lengths of the terrestrial arc comprehended between the parallels of Doddaguntah station, and the station near Patchipolliam.*

Stations at	Stations to	Bearings	Distances	Distances on the		Distance from Doddaguntah	
				Perpendicular	Meridian	Perpendicular	Meridian
Doddaguntah	Deorbetta	0° 13′ 08″,43 SE	135931,3	519,6 E	135930,3 S	519,6 E	135930,3 S
Deorbetta	Ponnasmalli	2 11 54,36 SE	174071,7	6677,6 E	173947,6 S	7197,2 E	309877,9 S
Ponnasmalli	Woorachmalli	3 49 54,39 SE	243502,4	16272,6 E	242958,2 S	23469,8 E	552836,1 S
Woorachmalli	Patchipolliam	7 53 51,52 SW	276169,4	24206,4 W	174498,5 S	736,6 W	7273346,6 S

TABLE II. *Length of the terrestrial arc comprehended between the parallels of Doddaguntah station and the station at Namthabad.*

Stations at	Stations to	Bearings	Distances	Distances on the		Distance from Doddaguntah	
				Perpendicular	Meridian	Perpendicular	Meridian
Deorbetta	Allasoor hill	0° 43′ 54″,55 NW	194662,8	2486,3 W	194646,9 N	1966,7 W	58716,6 N
Allasoor hill	Kulcottah hill	4 5 43,25 NW	94211,8	6728,3 W	93971,2 N	8695,0 W	152687,8 N
Kalcottah hill	Yerracondah	5 43 49,55 NE	180883,8	18060,9 E	179979,9 N	9365,9 E	332667,7 N
Yerracondah	Ooracondah	7 04 21,49 NW	126785,7	15610,8 W	125821,0 N	6244,9 W	458488,7 N
Ooracondah	Davurcondah	5 32 52,09 NE	150506,1	14550,4 E	149801,1 N	8305,5 E	608289,8 N
Davurcondah	Gooty Droog	0 16 40,56 NE	158946,2	771,1 E	158944,3 N	9076,6 E	767234,1 N
Gooty Droog	Namthabad	70 43 30,60 SW	16472,2	15548,9 W	5437,5S	6472,3 W	761796,6 N

TABLE III. *Length of the terrestrial arc between the parallels of Patchipolliam and the station near Punnæ.*

Stations at	Stations to	Bearings	Distances	Distances on the		Distance from Doddaguntah	
				Perpendicular	Meridian	Perpendicular	Meridian
Patchipolliam	Parteemalli	7° 56′ 29″,74 SW	120553,6	16656,1 W	119397,4 S	16656,1 W	119397,4 S
Parteemalli	Peermaulmalli	1 54 37 ,23 SW	133318,9	4444,2 W	133244,8 S	21100,3 W	252642,2 S
Peermaulmalli	Sudragherry	11 14 52 ,96 SE	207080,1	40392,4 E	203102,5 S	19212,1 E	455744,7 S
Sudragherry	Vulunkota	3 13 50 ,25 SW	330403,5	19127,1 W	338864,1 S	165,0 E	794608,8 S
Vulunkota	Kunnamopolha	0 10 35 ,28 SW	108363,9	333,8 W	108363,3 S	168,8 W	902972,1 S
Kunnamopolha	Punnæ station	0 27 21 ,16 SE	126132,4	1003,6 E	126128,4 S	834,8 E	1029100,5 S

TABLE IV. *Length of the terrestrial arc comprehended between the parallels of Doddaguntah and Daumergidda.*

Stations at	Stations to	Bearings	Distances	Distances on the		Distance from Doddaguntah	
				Perpendicular	Meridian	Perpendicular	Meridian
Gooty Droog	Koelcondah	10° 59′ 34″,9 NW	76750,9	15635,6 W	75342,5 N	5559,0 W	842576,6 N
Koelcondah	Poolycondah	4 06 33 ,9 NW	54084,1	3875,7 W	53945,0 N	9434,7 W	896521,6 N
Poolycondah	Kerrabellagal	13 21 56 ,9 NE	127920,1	29570,9 E	124455,3 N	20136,2 E	1020976,9 N
Kerrabellagal	Daroor hill	3 04 35 ,9 NW	150701,3	8088,4 W	150484,1 N	12047,8 E	1171461,0 N
Daroor hill	Inpahgut	0 45 15 ,7 NW	175159,1	2306,1 W	175144,0 N	9741,7 E	1346605,0 N
Inpahgut	Kotakodangul	1 42 38 ,8 NW	153863,3	4593,5 W	153794,7 N	5148,2 E	1500399,7 N
Kotakodangul	Shelapilly hill	2 20 25 ,7 NE	231767,9	9464,9 E	231574,5 N	14613,1 E	1731974,2 N
Shelapilly hill	Daumergidda	0 01 33 ,0 NE	103250,6	46,6 E	103250,6 N	14659,7 E	1835224,8 N

From the foregoing tables we collect the following particulars.

	Terrestrial arcs.	
	feet	fathoms
By Table I. the terrestrial arc between Doddaguntah and Patchipolliam is - - -	727334,6	
By Table II. The arc between Doddaguntah and Namthabad is - - - - -	761796,6	
Their sum will be the arc between Patchipolliam and Namthabad - - - =	1489131,2	248188,53
By Table III. the arc between Patchipolliam and Punnæ station is - - - -	1029100,5	171516,75
Their sum is the terrestrial arc between Punnæ and Namthabad - - - =	2518231,7	
By Table IV. the arc between Doddaguntah and Daumergidda is - - -	1835224,8	
From which subtract the arc between Doddaguntah and Namthabad - - - -	761796,6	
The difference will be the arc between Namthabad and Daumergidda - - - - =	1073428,2	178904,70
And the sum of the whole of these three sections will be the terrestrial arc between Punnæ and Daumergidda - - - -	- -	598609,98

Amplitudes of the celestial arc between the parallels of Punnæ and Patchipolliam.

Stars.	Zenith distances at		Amplitudes.
	Patchipolliam.	Punnæ.	
δ Hydræ	4° 37′ 12″,65 S	1° 47′ 01″,37 S	2° 50′ 11″,28
ε Hydræ	3 52 08 ,97 S	1 01 59 ,31 S	9 ,66
α Cancri	1 36 32 ,64 N	4 26 42 ,91 N	10 ,27
ο Leonis	0 13 18 ,16 S	2 36 52 ,07 N	10 ,23
Regulus	1 55 12 ,99 N	4 45 24 ,06 N	11 ,07
θ Leonis	5 29 54 ,26 N	8 20 03 ,44 N	9 ,18
β Leonis	4 39 59 ,40 N	7 30 11 ,59 N	12 ,19
ε Virginis	1 00 55 ,20 N	3 51 05 ,95 N	10 ,75
δ Serpentis	0 12 14 ,15 N	3 02 25 ,36 N	11 ,21
α Serpentis	3 56 48 ,46 S	1 06 38 ,10 S	10 ,36
α Herculis	3 37 38 ,58 N	6 27 48 ,35 N	9 ,77
α Ophiuchi	1 43 00 ,69 N	4 33 11 ,86 N	11 ,17
ε Aquilæ	2 35 16 ,44 N	5 25 29 ,25 N	12 ,81
ν Aquilæ	0 50 50 ,74 S	1 59 19 ,11 N	10 ,51
Atair	2 37 54 ,13 S	0 12 14 ,69 N	8 ,82
β Aquilæ	5 03 55 ,68 S	2 13 48 ,40 S	7 ,28
β Delphini	2 55 48 ,68 N	5 45 58 ,28 N	12 ,60
		Mean	2 50 10 ,55

Amplitude of the celestial arc between the parallels of Patchipolliam and Namthabad.

Stars.	Zenith distances at		Amplitudes.
	Patchipolliam.	Namthabad.	
ο Leonis	0°13′ 18″,16 S	4° 19′ 29″,91 S	4° 06′ 11″,75
Regulus	1 55 12 ,99 N	2 10 59 ,16 S	12 ,15
δ Leonis	5 29 54 ,26 N	1 23 42 ,08 N	12 ,18
β Leonis	4 39 59 ,40 N	0 33 49 ,17 N	10 ,23
ε Virginis	1 00 55 ,20 N	3 58 56 ,58 S	10 ,07
δ Serpentis	0 12 14 ,15 N	3 53 56 ,58 S	10 ,73
α Herculis	3 37 38 ,58 N	0 28 34 ,09 S	12 ,67
κ Ophiuchi	1 43 00 ,69 N	2 23 10 ,99 S	11 ,68
ε Aquilæ	2 35 16 ,44 N	1 30 53 ,62 S	10 ,06
γ Aquilæ	0 50 50 ,74 S	4 57 02 ,59 S	11 ,80
Atair	2 37 54 ,13 S	6 44 07 ,19 S	13 ,06
β Delphini	2 55 45 ,68 N	1 10 23 ,40 S	9 ,08
		Mean	4 06 11 ,28

Amplitude of the celestial arc between the parallels of Namthabad and Daumergidda.

Stars.	Zenith distances at		Amplitudes.
	Namthabad.	Daumergidda.	
ο Leonis	4° 19′ 30″,079 S	7°16′ 53″,233 S	2°57′ 23″,254
Regulus	2 10 59 ,463 S	5 08 21 ,582 S	22 ,099
γ Leonis	5 43 28 ,704 S	2 46 03 ,056 N	25 ,648
ζ Leonis	1 23 42 ,038 N	1 33 40 ,925 S	22 ,963
β Leonis	0 33 49 ,174 N	2 23 34 ,324 S	23 ,498
ε Virginis	3 05 14 ,750 S	6 02 36 ,570 S	21 ,820
η. Bootis	4 16 54 ,460 N	1 19 29 ,335 N	25 ,125
Arcturus	5 06 16 ,777 N	2 08 51 ,502 N	25 ,272
ε Bootis	0 31 35 ,868 S	3 28 55 ,050 S	19 ,182
δ Serpentis	3 53 56 ,290 S	6 51 19 ,536 S	23 ,246
β Serpentis	0 56 32 ,646 N	2 00 50 ,287 S	22 ,933
γ Serpentis	1 12 23 ,718 N	1 45 00 ,865 S	24 ,583
γ Herculis	4 31 19 ,103 N	1 33 55 ,469 N	23 ,634
		Mean	2 57 23 ,320

Amplitude of the whole celestial arc between the parallels of Punnæ and Daumergidda.

Stars.	Zenith distances at		Amplitudes.
	Punnæ.	Daumergidda.	
ο Leonis	2°36′ 51″,926 N	7°16′ 53″,233 S	9°53′ 45″,159
Regulus	4 45 23 ,979 N	5 08 21 ,582 S	45 ,561
θ Leonis	8 20 03 ,213 N	1 33 40 ,925 S	44 ,138
β Leonis	7 30 11 ,608 N	2 23 34 ,324 S	45 ,932
ε Virginis	3 51 06 ,083 N	6 02 36 ,570 S	42 ,653
δ Serpentis	3 02 25 ,643 N	6 51 19 ,536 S	45 ,179
γ Serpentis	8 08 47 ,269 N	1 45 00 ,865 S	48 ,134
		Mean	9 53 45 ,251

Latitude of Punnæ station deduced from the foregoing zenith distances of eight principal stars, whose declinations and annual variations are given in the Greenwich observations for 1802.

Stars.	For the beginning of 1805.		Latitudes.
	Mean declination.	Correct. zen. dist.	
Regulus	12°54′ 58″,93 N	4°45′ 24″,09 N	8°09′ 34″ ,84 N
β Leonis	15 39 45 ,28	7 30 11 ,59 N	33 ,70
α Serpentis	7 03 00 ,30	1 06 38 ,10 S	38 ,40
α Herculis	14 37 30 ,96	6 27 48 ,35 N	42 ,61
α Ophiuchi	12 42 50 ,91	4 33 11 ,86 N	39 ,05
γ Aquilæ	10 08 58 ,34	1 59 19 ,77 N	38 ,57
Atair	8 21 53 ,53	0 12 14 ,69 N	38 ,84
β Aquilæ	5 55 52 ,71	2 13 48 ,40 S	41 ,11
		Mean	8 09 38 ,39

Latitude of Punnæ station - -	8° 09′ 38″,39
Celestial arc between Punnæ and Patchipolliam	2 50 10 ,54
Their sum is the latitude of Patchipolliam	11 59 48 ,93

Celestial arc between Patchipolliam and Namthabad	4° 06′ 11″,28
Which added to the last gives the latitude of Namthabad	15 06 00 ,21
Celestial arc between Namthabad and Daumergidda	2 57 23 ,32
Which added gives the latitude of Daumergidda	18 03 23 ,53
The arc between Punnæ and Daumergidda by seven corresponding stars (9° 53′ 45″,25) added to the latitude of Punnæ (8° 09′ 38″, 39) gives	18 03 23 ,64
Mean of these two latitudes gives the correct latitude	18 03 23 ,58

By comparing the above three sections of the celestial arc with their respective terrestrial measures, we shall have the following conclusions.

Celestial arc between Punnæ and Patchipolliam	2° 50′ 10″,54
Latitude of the middle point (9° 34′ 43″, 6)	9 34 44
Terrestrial arc in fathoms	171516 ,75
Mean length of the degree due to latitude 9° 34′ 44″ in fathoms	60472 ,83
Celestial arc between Patchipolliam and Namthabad	4° 06′ 11″,28
Latitude of the middle point	13 02 55
Terrestrial arc in fathoms	248188,53
Mean degree due to latitude 13° 2′ 55″	60487,56
Celestial arc between Namthabad and Daumergidda	2° 57′ 23″,32

Terrestrial arc in fathoms - - 178904,70
Latitude of the middle point 16° 34′ 42″
Mean degree in fathoms due to latitude 16° 34′ 42″ 60512,78

So that by the above comparisons it appears that the
degree due to latitude - 9° 34′ 44″ is - 60472,83
the degree due to latitude 12° 2′ 55″ is - 60487,56
the degree due to latitude 16° 34′ 42″ is - 60512,78

It now remains to compare each of these mean degrees, *first* with the French measurement; *then* with the English; and *lastly* with the Swedish; and by proceeding on the elliptic theory, deduce from these data three mean ellipticities, and from these three *a general mean*, which must give nearly the true compression at the poles.

Previous to this determination, it will be necessary to investigate the requisite formulæ for obtaining the compression by a comparison of measured degrees in distant latitudes; and first, by the measured degrees on the meridian.

Let m' and m be the measured degrees, in latitudes $'l$ and l; and let a represent the equatorial diameter, and b the polar axis: that is, supposing the earth to be an ellipsoid, let a and b represent the transverse and conjugate axes of an elliptic meridian. Then it is known from conic sections and the nature of curvature, that $\frac{a^2 b^2}{2\,(\text{Cos.}^2\,'l.\,a^2 + \text{Sin.}^2\,'l.\,b^2)^{\frac{3}{2}}}$ is the radius of curvature of the ellipse at $'l$; and that $\frac{a^2 b^2}{2\,(\text{Cos.}^2\,l.\,a^2 + \text{Sin.}^2\,l.\,b^2)^{\frac{3}{2}}}$ is the radius of curvature of the same, or any other elliptic meridian, on the same ellipsoid, at the point l. And since the degrees at $'l$ and l are as the radii of curvature at these points, we have $m' : m :: \frac{a^2 b^2}{2\,(\text{Cos.}^2\,'l.\,a^2 + \text{Sin.}^2\,'l.\,b^2)^{\frac{3}{2}}} : \frac{a^2 b^2}{2\,(\text{Cos.}^2\,l.\,a^2 + \text{Sin.}^2\,l.\,b^2)^{\frac{3}{2}}}$

$:: (\text{Cos.}^2\,{}'l.\ a^2 + \text{Sin.}\ {}'l.\ b^2)^{-\frac{3}{2}} : (\text{Cos.}^2\,l.\ a^2 + \text{Sin.}^2\,l.\ b^2)^{-\frac{3}{2}}$. Now to simplify this expression, if $a = 1$, e the ellipticity; and therefore $b = 1 - e$, and $b^2 = 1 - 2\,e$ nearly, because e^2 must be very small. Then will $m' : m :: (\text{Cos.}^2\,{}'l + (1\ 2e).\ \text{Sin.}^2\,{}'l)^{-\frac{3}{2}} : (\text{Cos.}^2\,l + (1 - 2\,e).\ \text{Sin.}^2\,l)^{-\frac{3}{2}}$. And if $(1 - \text{Sin.}^2\,{}'l)$ and $(1 - \text{Sin.}^2\,l)$ be substituted for $\text{Cos.}^2\,{}'l$ and $\text{Cos.}^2\,l$. the expression will be transformed into $(1 - 2\,e.\ \text{Sin.}^2\,{}'l)^{-\frac{3}{2}}$ and $(1\ 2e.\ \text{Sin.}^2\,l)^{-\frac{3}{2}}$; or $1 + 3e.\ \text{Sin.}^2\,{}'l$ and $1 + 3e.\ \text{Sin.}^2\,l$ nearly, by developing the series, and leaving out all the quantities involving e^2 or its higher powers. Hence $m' : m :: 1 + 3e.\ \text{Sin.}^2\,{}'l : 1 + 3e.\ \text{Sin.}^2\,l$. (1)
which reduced gives $e = \dfrac{m' - m}{3\,(m.\ \text{Sin.}^2\,{}'l - m'.\ \text{Sin.}^2\,l)}$. (2)
and when the degrees are contiguous, or very near to each other, this expression may be rendered still more simple by making $m' = m$ in the denominator, which under these circumstances will scarcely affect the result: whence

$$e = \frac{m' - m}{3m\,(\text{Sin.}^2\,{}'l - \text{Sin.}^2\,l)}. \quad (3)$$

By this expression it will be easy to estimate the increments to degrees lying contiguous to each other: for if m, m', m'', &c. be contiguous degrees in latitudes l; $'l$, $''l$, &c. that is l, $l + 1°$; $l + 2°$ &c. Then we shall have $e = \dfrac{m'' - m}{3(\text{Sin.}^2\,{}''l - \text{Sin.}^2\,{}'l)}$; which being made equal to $\dfrac{m' - m}{3\,(\text{Sin.}^2\,{}'l - \text{Sin.}^2 l)}$ and reduced, we get $m' - m : m'' - m :: \text{Sin.}^2\,{}'l - \text{Sin.}^2\,l : \text{Sin.}^2\,{}''l - \text{Sin.}^2\,l.$; and in like manner $m'' - m : m''' - m :: \text{Sin.}^2\,{}''l - \text{Sin.}^2\,l : \text{Sin.}^2\,{}'''l - \text{Sin.}^2\,l$; and $m''' - m : m'''' - m :: \text{Sin.}^2\,{}'''l - \text{Sin.}^2\,l : \text{Sin.}^2\,{}''''l - \text{Sin.}^2\,l.$ &c. from which it appears that the increments to the degrees, beginning with the lowest latitude, will always be as the increments to the squares of the sines of the corresponding latitudes: and if m be at the equator where the Sin. l is o, then we shall have $m' - m : m'' - m :: \text{Sin.}^2\,{}'l : \text{Sin.}^2\,{}''l$.

Since by equation 1, $m' : m :: 1 + 3e.\,\text{Sin.}^2\,{}'l : 1 + 3e.\,\text{Sin.}^2\,l$; then $m' = m\left(\frac{1 + 3e.\,\text{Sin.}^2\,{}'l}{1 + 3e.\,\text{Sin.}^2\,l}\right)$ . . . . . . (4)

$$\text{and } m = m'\left(\frac{1 + 3e.\,\text{Sin.}^2\,l}{1 + 3e.\,\text{Sin.}^2\,{}'l}\right) \quad . \ . \ . \ . \quad (5)$$

When m is at the equator, and therefore Sin. $l = o$; then

$$m' = m\,(1 + 3e.\,\text{Sin.}^2\,{}'l) \quad . \ . \ . \quad (6)$$

If m' be at the pole, and therefore $\text{Sin.}^2\,{}'l = 1$, then we

$$\text{have } m' = m\left(\frac{1 + 3e}{1 + 3e.\,\text{Sin.}^2\,l}\right) \quad . \ . \quad (7)$$

If the degrees perpendicular to the meridian be made use of, let p' and p be the measures of those degrees in latitudes ${}'l$ and l, then the radius of curvature of the perpendicular degree at ${}'l$ being as $\frac{1}{2(1 - 2e.\,\text{Sin.}^2\,{}'l)^{\frac{1}{2}}} = \frac{1}{2}(1 - 2e.\,\text{Sin.}^2\,{}'l)\tfrac{1}{2} =$ $= \frac{1}{2}\,(1 + e.\,\text{Sin.}^2\,{}'l)$ very nearly; and for the same reason the radius of curvature of the perpendicular degree at l will be as $\frac{1}{2}\,(1 + e.\,\text{Sin.}^2\,l)$ very nearly; so that we get $p' : p :: 1 + e.\,\text{Sin.}^2\,{}'l : 1 + e.\,\text{Sin.}^2\,l$ . . . . . (8)

$$\text{and when reduced gives } e = \frac{p' - p}{p.\,\text{Sin.}^2\,{}'l - p'\,\text{Sin.}^2\,l} \quad . \ . \ . \quad (9)$$

$$\text{From equation 8, } p' = p\left(\frac{1 + e.\,\text{Sin.}^2\,{}'l}{1 + e.\,\text{Sin.}^2\,l}\right) \quad . \ . \ . \quad (10)$$

$$\text{and } p = p\left(\frac{1 + e.\,\text{Sin.}^2\,l}{1 + e.\,\text{Sin.}^2\,{}'l}\right) \quad . \ . \ . \quad (11)$$

If p be on the equator where the Sin. l vanishes, then equation 10 becomes $p' = p\,(1 + e.\,\text{Sin.}^2\,{}'l)$ . . . (12)

If p' be at the pole, and therefore the cosine of ${}'l$ unity, then equation 10 becomes $p' = p\left(\frac{1 + e}{1 + e.\,\text{Sin.}^2\,l}\right)$ . . . (13)

Since the degree on the meridian, and the degree perpendicular to the meridian, are equal at the pole, we shall have by equations 7, and 13, $p\left(\frac{1+e}{1+e.\,\text{Sin.}^2 l}\right)=m\left(\frac{1+3e}{1+3e.\,\text{Sin.}^2 l}\right)$, where p and m are in the same latitude l. Now $m\left(\frac{1+3e}{1+3e.\,\text{Sin.}^2 l}\right)=m(1+3e).(1+3e.\,\text{Sin.}^2 l)^{-1}=$ $=m(1+3e.\,\text{Cos.}^2 l)$ nearly; and therefore $p\left(\frac{1+e}{1+e\,\text{Sin.}^2 l}\right)=$ $=p(1+e).(1+e.\,\text{Sin.}^2 l)^{-1}=p(1+e.\,\text{Cos.}^2 l)$ nearly. Hence $m(1+3e.\,\text{Sin.}^2 l)=p(1+e.\,\text{Cos.}^2 l)$ which reduced gives $e=\frac{p-m}{(3m-p)\,\text{Cos.}^2 l}$ . . . . (14)

Since $m(1+3e.\,\text{Cos.}^2 l)=p(1+e.\,\text{Cos.}^2 l)$, we get

$m:p::1+e\,\text{Cos.}^2 l:1+3e\,\text{Cos.}^2 l$ . . (15)

and when $l=o$, and its Cos. equal 1, then $m:p::1+e:1+3e$ . . . . (16)

If we make use of the degrees of longitude, then let d' and d represent their measures in latitudes ${}'l$ and l; and their respective radii of curvature at ${}'l$ and l will be expressed by $\frac{\text{Cos.}\,{}'l}{2(\text{Cos.}^2{}'l+(1-2e)\,\text{Sin.}^2{}'l)^{\frac{1}{2}}}$ and $\frac{\text{Cos.}\,l}{2(\text{Cos.}^2 l+(1-2e).\,\text{Sin.}^2 l)^{\frac{1}{2}}}$, and therefore $d':d::\frac{\text{Cos.}\,{}'l}{(\text{Cos.}^2{}'l+(1-2e)\,\text{Sin.}^2{}'l)^{\frac{1}{2}}}:\frac{\text{Cos.}\,l}{(\text{Cos.}^2 l+(1-2e)\,\text{Sin.}^2 l)^{\frac{1}{2}}}$; that is $d':d::\frac{\text{Cos.}\,{}'l}{(1-2e.\,\text{Sin.}^2{}'l)^{\frac{1}{2}}}:\frac{\text{Cos.}\,l}{(1-2e.\,\text{Sin.}^2 l)^{\frac{1}{2}}}$; that is $d':d::$ $\text{Cos.}\,{}'l(1-2e.\,\text{Sin.}^2{}'l)^{-\frac{1}{2}}:\text{Cos.}\,l\,(1-2e.\,\text{Sin.}^2 l)^{-\frac{1}{2}}$; that is $d':d::\text{Cos.}\,{}'l\,(1+e.\,\text{Sin.}^2{}'l):\text{Cos.}\,l\,(1+e.\,\text{Sin.}^2 l)$ . . (17)

and this reduced gives $e=\frac{d'\,\text{Cos.}\,l-d.\,\text{Cos.}\,{}'l}{d.\,\text{Cos.}\,{}'l.\,\text{Sin.}^2{}'l-d'.\,\text{Cos.}\,l.\,\text{Sin.}^2 l}$ (18)

since $d':d::\text{Cos.}\,{}'l\,(1+e.\,\text{Sin.}^2{}'l):\text{Cos.}\,l\,(1+e.\,\text{Sin.}^2 l)$; if d be at the equator where Sin. l vanishes; then

$d:d'::1:\text{Cos.}\,{}'l(1+e.\,\text{Sin.}^2{}'l)$ . . . . (19)

From this equation we get $d = \frac{d'}{\text{Cos.}\,'l\,(1 + e.\,\text{Sin.}^2\,'l)}$.

And from equation 9 we get $p = \frac{p'}{1 + e.\,\text{Sin.}^2\,l}$; and since at the equator the degree of longitude, and the perpendicular degree are equal, then $\frac{d'}{\text{Cos.}\,'l\,(1 + e.\,\text{Sin.}^2\,'l)} = \frac{p'}{1 + e.\,\text{Sin.}^2\,'l}$, and this reduced we shall have $d' = p'.\,\text{Cos.}\,'l$, where d' and p' are in the same latitude. Hence $d' : p' :: \text{Cos.}\,'l : \text{rad}$ . . . (20)

I shall now proceed to determine the compression at the poles from the foregoing three sections of the arc; and comparing each, *first* with the French degree in latitude 47°30′46″, equal 60779 fathoms; *then* with the English degree in latitude 52° 2′ 20″, which is 60820 fathoms; and *lastly* with the Swedish degree in latitude 66° 20′ 12,″ when it is 60955 fathoms.

With respect to the French degree, as there appears much irregularity in the different sections of the arc between Dunkirk and Montjouy, I have used *that* between the Pantheon at Paris, and Eveaux, as given by DE LAMBRE in the *Base du Systeme Mètrique*, as it appears to be the most consistent. This degree is 57066 toises, equal to 60798 fathoms. But their measurements being all reduced to the temperature of 32° of FAHRENHEIT'S thermometer, the above degree will require a deduction of 19 fathoms nearly, to make it what it would have measured by the brass standard at the temperature of 62°, which is our standard temperature. Hence, 60779 fathoms is the degree in latitude 47° 30′ 46″ to compare with the Indian measurements.

Let this degree be denoted by m'; and its latitude, 47° 30′ 46″ by $'l$; and let the degree 60472,83 fathoms be m, and its latitude 9° 34′ 44″ be l. Then by equation 1,

$e = \frac{m' - m}{3(m.\ \text{Sin.}^2\,'l - m'.\ \text{Sin.}^2\,l)}$. We shall then have the type of calculation as follows:

$l = 9^\circ\ 34'\ 44''$ $\qquad m = 60472{,}83$

$'l = 47\ \ 30\ \ 46$ $\qquad m' = 60779$

$m' - m = 306{,}17$

$\text{Log.}\ m = 4{,}7815603$

$\text{log.}\ (\text{Sin.}^2\,'l) = \bar{1}{,}7354392$

$4{,}5169996.$ its nat. no. $= 32885{,}1$

$\text{Log.}\ m' = 4{,}7837536$

$\text{log.}(\text{Sin.}^2 l) = \bar{2}{,}4423288$

$3{,}2260824.$ nat no. . $1683{,}0$

$31202{,}1 = (m\ \text{S.}^2\,'l - m'\ \text{S.}^2\,l)$

3

Hence $e = \frac{306{,}17}{93606{,}3} = \frac{1}{305{,}73}$ $\quad 93606{,}3$

Let $l = 13^\circ\ \ 2'\ 55''$ . . $m = 60487{,}56$

$'l = 47\ \ 30\ \ 46$ . . $m' = 60779$

$291{,}44 = m' - m$

$\text{Log.}\ m = 4{,}7816660$

$\text{log.}\ (\text{Sin.}^2\,'l) = \bar{1}{,}7354392$

$4{,}5171052$ . . nat. no. $= 32893{,}2$

$\text{Log.}\ m' = 4{,}7837536$

$\text{log.}\ (\text{Sin.}^2\,l) = \bar{2}{,}7073618$

$3{,}4911154$ . . nat. no. $= 3098{,}3$

$29794{,}9$

3

$89384{,}7$

Hence $e = \frac{291{,}44}{89384{,}7} = \frac{1}{306{,}70}$

$$\text{Let } l = 16^\circ\ 34'\ 42'' \ldots m = 60512{,}78$$

$$'l = 47\ \ 30\ \ 46 \ \ldots m' = 60779 \ldots$$

$$m' - m = 266{,}22$$

$$\text{Log. } m = 4{,}7818471$$

$$\text{log. } (\text{Sin.}^2\, 'l\,) = \bar{1}{,}7354392$$

$$4{,}5172463 \ldots \text{nat. no.} = 32903{,}8$$

$$\text{Log. } m' = 4{,}7837536$$

$$\text{log. } (\text{Sin.}^2\, l) = \bar{2}{,}9106824$$

$$3{,}6944360 \ldots \text{nat. no.} \ldots 4948{,}1$$

$$27955{,}7$$

$$3$$

$$83867{,}1$$

$$\text{Hence } e = \frac{266{,}22}{83867{,}1} = \frac{1}{315{,}03}$$

Whence the mean of $\frac{1}{305{,}73}$; $\frac{1}{306{,}7}$; $\frac{1}{315{,}03} = \frac{1}{309{,}15}$; or the mean compression deduced from the mean degrees given by these three sections, compared with the French measure.

If we proceed in the same manner with the English and Swedish measures, we shall have by the whole as follows:

By the French $\frac{1}{305{,}73}$; $\frac{1}{306{,}7}$; $\frac{1}{315{,}03}$; mean $\frac{1}{309{,}15}$

By the English $\frac{1}{310{,}28}$; $\frac{1}{311{,}36}$; $\frac{1}{318{,}97}$; mean $\frac{1}{313{,}54}$

By the Swedish $\frac{1}{305{,}14}$; $\frac{1}{305{,}72}$; $\frac{1}{310{,}72}$; mean $\frac{1}{307{,}19}$

And the mean of the three means equal $\frac{1}{309{,}96} = \frac{1}{310{,}00}$ nearly for the compression at the poles, as deduced from these comparisons; which compression will be adopted for computing the different degrees from the equator to the pole.

All this is supposing the earth to be an ellipsoid. But that these Indian measurements may rest on their own ground, I shall examine whether the increments to a succession of contiguous degrees as deduced from the present data, be consistent with the elliptic hypothesis, beginning with the degree in latitude 9° 34′ 44″, as determined by observation. To effect this let $m^{(1)}$, $m^{(2)}$, $m^{(3)}$, &c. be the measures of *complete* contiguous degrees on the meridian in latitudes $l^{(1)}$, $l^{(2)}$, $l^{(3)}$, &c. Then, if a meridian of the earth be an ellipse, we know from equation 2, that the compression will be expressed by $\frac{m^{(2)}-m^{(1)}}{3\,(\text{Sin.}^2\, l^2-\text{Sin.}^2\, l^{(1)}}$; or $\frac{m^{(3)}-m^{(1)}}{3\,(\text{Sin.}^2\, l^{(3)}-\text{Sin.}^2\, l^{(1)}}$; or $\frac{m^{(n)}-m^{(1)}}{3\,(\text{Sin.}^2\, l^{(n)}-\text{Sin.}^2\, l^{(1)}}$; let the length of the diameters be what they will. So that we shall have $\frac{m^{(2)}-m^{(1)}}{3\,m^{(1)}\,(\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)})}$; $=$

$$=\frac{m^{(3)}-m^{(1)}}{3\,m'\,(\text{Sin.}^2\, l^{(3)}-\text{Sin.}^2\, l^{(1)})} : \text{or } \frac{m^{(2)}-m^{(1)}}{3\,m^{(1)}\,(\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)})} ; =$$

$$=\frac{m^{(3)}-m^{(1)}}{3\,m^{(1)}\,(\text{Sin.}^2\, l^{(3)}-\text{Sin.}^2\, l^{(1)})} ;$$ and by reduction $m^{(3)}-m^{(1)} =$

$$=(m^{(2)}-m^{(1)}).\frac{\text{Sin.}^2\, l^{(3)}-\text{Sin.}^2\, l^{(1)}}{\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)}}$$

and $m^{(3)}=m^{(1)}+(m^{(2)}-m^{(1)}).\frac{\text{Sin.}^2\, l^{(3)}-\text{Sin.}^2\, l^{(1)}}{\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)}}$

$$m^{(4)}=m^{(1)}+(m^{(2)}-m^{(1)}).\frac{\text{Sin.}^2\, l^{(4)}-\text{Sin.}^2\, l^{(1)}}{\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)}}$$

$$m^{(5)}=m^{(1)}+(m^{(2)}-m^{(1)}).\frac{\text{Sin.}^2\, l^{(5)}-\text{Sin.}^2\, l^{(1)}}{\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)}},$$ &c. to

to $m^{(n)}=m^{(1)}+(m^{(2)}-m^{(1)}).\frac{\text{Sin.}^2\, l^{(n)}-\text{Sin.}^2\, l^{(1)}}{\text{Sin.}^2\, l^{(2)}-\text{Sin.}^2\, l^{(1)}}$

Also $m^{(2)} = m^{(1)} + (m^{(2)} - m^{(1)}) . \frac{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}}$; that is $m^{(2)} = m^{(1)} + (m^{(2)} - m^{(1)})$, by preserving the expression $m^{(2)} - m^{(1)}$, which we will call d. Then we shall have

$$m^{(1)} = m^{(1)} + o$$

$$m^{(2)} = m^{(1)} + d$$

$$m^{(3)} = m^{(1)} + d \left\{ \frac{\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)}}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}} \right\}$$

$$m^{(4)} = m^{(1)} + d \left\{ \frac{\text{Sin.}^2 l^{(4)} - \text{Sin.}^2 l^{(1)}}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}} \right\}, \text{ \&c.}$$

$$\text{to } m^{(n)} = m^{(1)} + d \left\{ \frac{\text{n.}^2 l^{(n)} - \text{Sin.}^2 l^{(1)}}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}} \right\}$$

Here d is the only unknown quantity to be determined, since $m^{(1)} + m^{(2)} + m^{(3)} \ldots . m^{(n)} = \text{A}$. the terrestrial measure of the arc of n complete degrees; $m^{(1)}$ being the measure of the first degree in latitude $l^{(1)}$ by observation.

$$\text{Then A} = nm^{(1)} + d \left\{ o + 1 + \frac{\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)} \ldots . \text{Sin.}^2 l^{(n)} - \text{Sin.}^2 l^{(1)}}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}} \right\}$$

$$\text{And } d = \left\{ \frac{(\text{A} - nm^{(1)}) . (\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)})}{(\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}) + (\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)}) + \ldots . \text{Sin.}^2 l^{(n)} - \text{Sin.}^2 l^{(1)})} \right\}$$

when d becomes a known quantity. And since $\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}$ is a constant and known quantity, if $\frac{d}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}}$ be called Q, we shall have the order of contiguous degrees as follows:

$$m^{(1)} = m + o$$

$$m^{(2)} = m + d$$

$$m^{(3)} = m + \text{Q} \left\{ \text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)} \right\}$$

$$m^{(4)} = m + Q\left\{\text{Sin.}^{2} l^{4)} - \text{Sin.}^{2} l^{(1)}\right\} \&\text{c.}$$

$$\text{to } m^{(n)} = m + Q\left\{\text{Sin.}^{2} l^{(n)} - \text{Sin.}^{2} l^{(1)}\right\}$$

To apply this formula to the present measurement, it will be necessary to have a terrestrial arc to correspond with the celestial one of complete degrees, and the first degree determined by observation. If we begin with the degree in latitude 9° 34′ 44″, which is 60472,83 fathoms, as the mean degree deduced from an arc of 2° 50′ 10″, 54, where the

fathoms.

corresponding terrestrial arc, is - - 171516,75

The half of which is the distance of the middle point of the arc from Patchipolliam, equal - 85758,375

To which add half the degree south, or - 30236,415

Their sum is the terrestrial arc between half the degree south of the middle point and Patchipolliam, - - - = 115994,790

The latitude of whose commencement is 9°4′33″,66 the latitude of the south extremity of an arc of complete degrees.

Now the terrestrial arc between Patchipolliam and Namthabad is - - - 248188,534

And between Namthabad and Daumergidda - 178904,700

Their sum is the terrestrial arc between 9° 34′ 43″ 66 and Daumergidda - - 543088,024

The latitude of Daumergidda, by adding the arc between Namthabad and Daumergidda (=2° 57′ 23″,32) by 13 stars, to the latitude of Namthabad (15° 6′ 0″,21) gives - 18°03′ 23″,64

The latitude of Daumergidda, by adding the whole arc between Punnæ and Daumergidda

($9^\circ\ 53'\ 45'',25$) as determined by seven corresponding stars, to the latitude of Punnæ, is $18^\circ\ 03'\ 23'',64$

The mean of which, or correct latitude, is $18\ \ 03\ \ 23\ ,58$

Hence from $18^\circ\ \ 3'\ \ 23'',58$

Subtract $9\ \ \ 4\ \ 43\ ,66$

Dif. or arc $8\ \ 58\ \ 39\ ,92$ whose measure is $543088,024$

To which add $0\ \ \ 1\ \ 20\ ,08$ whose measure is $1345,184$

$\left.\begin{array}{l}\text{Gives the N}^\circ.\ n\ \text{of}\\ \text{complete degrees}\end{array}\right\} = 9^\circ\ 0'0''$ whose measure (A) is $544433,21$

Now the measure of the first degree is - $60472,83$

And $n = 9$, therefore $n\,m^{(1)}$ is equal - $544255,47$

Which subtracted from A gives $A - nm^{(1)} = 117,74$

And $\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)} = ,006014$; therefore, $006014 \times 117,74 = 1,0689284 = (A - n\ m^{(1)}) \cdot (\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)})$ the numerator; and the denominator $(\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}) + {}$ $+ (\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)}) + (\text{Sin.}^2 l^{(4)} - \text{Sin.}^2 l^{(1)}) + \ldots$ $(\text{Sin.}^2 l^{(9)} - \text{Sin.}^2 l^{(1)})$ is equal $0,263137$. Hence we shall have

$$\frac{(A - nm^{(1)}) \cdot (\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)})}{(\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}) + (\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)}) + \ldots (\text{Sin.}^2 l^{(9)} - \text{Sin.}^2 l^{(1)})},\ \text{equal}$$

$$\frac{1,0689284}{0,263137} = 4,06225 = d;\ \text{and} \frac{d}{\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}}$$

$= \frac{4,06225}{,006014} = 675,47 = Q$, and from these the following table has been constructed.

TABLE I.

	Degrees.	Latitudes.		
		°	′	″
$m^{(1)} = m^{(1)} + o$	60472,83	9	34	44
$m^{(2)} = m^{(1)} + d$	60476,89	10	34	44
$m^{(3)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)})$	60481,34	11	34	44
$m^{(4)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(4)} - \text{Sin.}^2 l^{(1)})$	60486,16	12	34	44
$m^{(5)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(5)} - \text{Sin.}^2 l^{(1)})$	60491,36	13	34	44
$m^{(6)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(6)} - \text{Sin.}^2 l^{(1)})$	60496,92	14	34	44
$m^{(7)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(7)} - \text{Sin.}^2 l^{(1)})$	60502,85	15	34	44
$m^{(8)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(8)} - \text{Sin.}^2 l^{(1)})$	60509,12	16	34	44
$m^{(9)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(9)} - \text{Sin.}^2 l^{(1)})$	60515,74	17	34	44
Sum -	544433,21	= A		

According to this table, the degree in latitude 16° 34′ 44″ is 60509,12 fathoms; and the mean degree for latitude 16° 34′ 42″, as deduced from the arc between Namthabad and Daumergidda is 60512,78 fathoms, which exceeds the computed one for latitude 16° 34′ 44″ (which may be considered the same) only 3,66 fathoms.

It may however be necessary to notice that any one of the expressions $\frac{m^{(2)} - m^{(1)}}{3\,(\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)})}$; $\frac{m^{(3)} - m^{(1)}}{3\,(\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)})}$, &c. will bring out a compression equal $\frac{1}{269}$ nearly, which differs considerably from the general mean. But a very small difference in the numerator will produce a great difference in the compression.

If we suppose $\frac{1}{310}$ to be the true compression, let it be determined what the value of $m^{(1)}$ ought to be to bring out that compression; and by that means to detect the errors of the observed degrees, in latitude 9° 34′ 44″, and 16° 34′ 42″, which last may be compared with $m^{(8)}$.

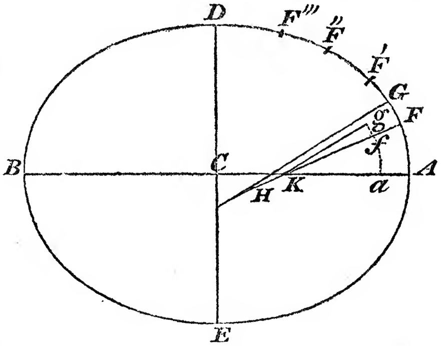
Put $A = 544433,21$; $a = \text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)} = ,006014$, radius being unity; $b = (\text{Sin.}^2 l^{(2)} - \text{Sin.}^2 l^{(1)}) + (\text{Sin.}^2 l^{(3)} - - \text{Sin.}^2 l^{(1)}) + \ldots\ldots (\text{Sin.}^2 l^{(9)} - \text{Sin.}^2 l^{(1)}) = ,263137$. Then $d = (m^{(2)} - m^{(1)}) = \frac{(A - nm^{(1)}) . a}{b}$; and $\frac{d}{3\,m^{(1)} . a} = = \frac{A - n\,m^{(1)}}{3\,b . m^{(1)}} = \frac{1}{310}$; from which is deduced $m^{(1)} = \frac{310 . A}{3\,b + 310 . n} = = 60475,47$ fathoms. Whence $d = \frac{(A - nm^{(1)}) . ,006014}{,263137} = 3,5192$. And $Q = \frac{d}{,006014} = 585,17$. From these the following table has been computed, from which it appears that the first degree by measurement, is 2,6 fathoms in defect; and that the one in latitude 16° 34′ 42″ is 5,89 fathoms in excess; either of which is too small to affect the elliptic hypothesis; the greatest being only $\frac{1}{3}$ of a second on the earth's surface.

TABLE II.

	Degrees.	Latitudes.		
		°	′	″
$m^{(1)} = m^{(1)} + o$ - -	60475,47	9	34	44
$m^{(2)} = m^{(1)} + d$ - - -	60478,99	10	34	44
$m^{(3)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(3)} - \text{Sin.}^2 l^{(1)})$	60482,84	11	34	44
$m^{(4)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(4)} - \text{Sin.}^2 l^{(1)})$	60487,02	12	34	44
$m^{(5)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(5)} - \text{Sin.}^2 l^{(1)})$	60491,53	13	34	44
$m^{(6)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(6)} - \text{Sin.}^2 l^{(1)})$	60496,34	14	34	44
$m^{(7)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(7)} - \text{Sin.}^2 l^{(1)})$	60501,47	15	34	44
$m^{(8)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(8)} - \text{Sin.}^2 l^{(1)})$	60506,91	16	34	44
$m^{(9)} = m^{(1)} + Q\,(\text{Sin.}^2 l^{(9)} - \text{Sin.}^2 l^{(1)})$	60512,64	17	34	44
Sum	544433,21	= A		

From inspecting these two tables, it appears that the degree in latitude 13° 34′ 44″ is nearly the same in each, and the mean is 60491,46 fathoms; which certainly must be near the truth. I shall therefore adopt it with the compression $\frac{1}{310}$ for computing the general tables of degrees for every third degree of latitude from the equator to the pole.

With respect, however, to the compression, that nothing may be left undone to give full and entire satisfaction on this subject, I shall here add an investigation similar to that given by Professor PLAYFAIR, in the 5th vol. of the Edinburgh Philosophical Transactions, where, in place of using the measures of degrees due to particular latitudes, two measured arcs of large amplitudes are made use of, the latitudes of whose extremities have been determined with great accuracy.



Let ADBE be a meridian of the earth, where A is at the equator, and D at the pole. Suppose F to be any point on that meridian, and FH the radius of curvature of the ellipse at that point. Put A C $= a$; DC $= b$; c being the centre of the ellipse; and let A be equal the angle AKF, the latitude of F; or let it be the measure of the arc of latitude aKf, to radius unity, or aK: that is, the *measure* of the angle aKf in parts of the radius aK, or unity. Let GF be an indefinitely small part of the ellipse. Then if AF $= z$, GH $= \dot{z}$ the fluxion of the arc AF of the ellipse; and if GH be drawn, then the angle GHF $= < g$ K$f = \dot{A}$ or fg the fluxion of the arc of latitude af to radius 1. Hence as $1 : \dot{A} :: \text{FH} : \dot{z} = \dot{A}.\text{FH}$. But the radius of curvature FH $= a^2 b^2 (a^2 - a^2. \text{Sin.}^2 \text{A} + b^2 \text{Sin. A})^{-\frac{3}{2}}$. Let e be the ellipticity, or $a - b$; then $b = a - e$, and $b^2 = a^2 - 2\,ae$ very nearly, since e^2 is very small. Hence FH $= a^3 (a - 2e) . (a^2 - 2ae. \text{Sin.}^2 \text{A})^{-\frac{3}{2}}$. But $(a^2 - - 2\,ae. \text{Sin.}^2 \text{A})^{-\frac{3}{2}} = (a^2)^{-\frac{3}{2}}. (1 - \frac{2e}{a}. \text{Sin.}^2 \text{A})^{-\frac{3}{2}} = = a^{-3}. (1 + \frac{3e}{a}. \text{Sin.}^2 \text{A})$ very nearly, by rejecting all the terms involving e^2 and its higher powers. Hence FH $= = a^3 (a - 2e). a^{-3}. (1 + \frac{3e}{a}. \text{Sin.}^2 \text{A}) = a - 2e + 3e. \text{Sin.}^2 \text{A}$, which substituted for FH, we get $\dot{z} = \dot{A}(a - 2e + 3e. \text{Sin.}^2 \text{A}) = = \dot{A} (a - 2e) + \dot{A} (3e. \text{Sin.}^2 \text{A})$. But $\text{Sin.}^2 \text{A} = \frac{1 - \text{Cos. } 2\text{A}}{2}$, and therefore $\dot{z} = \dot{A} (a - 2e) + \frac{3}{2} e \dot{A} - \frac{3}{2} e \dot{A}. \text{Cos. } 2\text{A}$; whose fluent is $z = (a - \frac{3}{4} e) \text{A} + \frac{3}{4} e. \text{Sin. } 2\text{A} = a\text{A} - e (\frac{\text{A}}{2} + \frac{3}{4} \text{A.Sin.} 2\text{A})$ which requires no correction. And this is the measure of an arc on the meridian extending from the equator to the latitude of the point **F**; where A denotes the arc of latitude in parts of the radius 1.

Let F′ be any other point on the meridian, whose arc of latitude is A′. Then $AF' = a\,A' - e\left(\frac{A'}{2} + \frac{3}{4}\,A'.\ \text{Sin.}\ 2A\right)$ and therefore $F\,F' = a(A' - A) - e\left\{\frac{A' - A}{2} + \frac{3}{4}.\ \text{Sin.}\ 2A' - \frac{3}{4}.\ \text{Sin.}\ 2A.\right\}$

Let F″, F‴, be any other two points on the meridian whose respective arcs of latitude are A″ and A‴. Then from the same reasoning as above, we have $F''\,F''' = a\,(A''' - A'') - - e\left\{\frac{A''' - A''}{2} + \frac{3}{4}.\ \text{Sin.}\ 2\,A''' - \frac{3}{4}\ \text{Sin.}^{2}\,A''\right\}$

Now FF′ and F″ F‴ are here supposed to be measured arcs on the meridian, whose respective lengths in fathoms may be called L and L′, corresponding with the celestial arcs A′—A, and A‴ — A″. To shorten the operation, put $A' - A = r$; $A''' - A'' = r'$. Also $\frac{A' - A}{2} + \frac{3}{4}\ \text{Sin.}\ A' - \frac{3}{4}.\ \text{Sin.}\ A = S$, and $\frac{A''' - A''}{2} + \frac{3}{4}.\ \text{Sin.}\ 2\,A''' - \frac{3}{4}.\ \text{Sin.}\ 2\,A'' = S'$. Then we have $L = ar - es$; $L' = ar' - es'$. And therefore $a = \frac{s'L - sL'}{rs' - r's}$; $- - e = \frac{r'L - rL'}{rs' - r's}$; and $\frac{e}{a} = \frac{r'L - rL'}{s'L - sL'}$ equal the compression expressed in fractional parts of the semi-equatorial diameter.

To apply this to the case in question,

	°	′	″	
Let A = the latitude of Punnæ =	8	9	38,4	
A = the lat. of Daumergidda =	18	3	23,6	
$A' - A = r$ - -	9	53	45,2	=,1727158
A'' = lat. of Montjouy -	41	21	44,96	
A''' = lat. of Dunkirk -	51	02	09,2	
$A''' - A'' = r'$ - -	9	40	24,24	=,1688327

$$\text{Put } s = \frac{A' - A}{2} + \tfrac{3}{4}\,\text{Sin.}\,2\,A' - \tfrac{3}{4}\,.\,\text{Sin.}\,2\,A = ,3176258$$

$$s' = \frac{A''' - A'}{2} + \tfrac{3}{4}\,\text{Sin.}\,2\,A''' - \tfrac{3}{4}\,.\,\text{Sin.}\,2\,A'' = ,0738689$$

$$\left.\begin{array}{l} L = 598610 \\ {}^{*}L' = 587475{,}41 \end{array}\right\}\begin{array}{l}\text{arc be-}\\ \text{tween}\end{array}\left\{\begin{array}{l}\text{Punnæ and Daumergidda,}\\ \text{Montjouy and Dunkirk.}\end{array}\right.$$

$$a = \frac{s'\,L - s\,L'}{rs' - r's} = 3483955$$

$$e = \frac{r'\,L - s\,L'}{rs' - r's} = 9820{,}8$$

$$\frac{e}{a} = \frac{r'\,L - r\,L'}{s'\,L - sL'} = \frac{1}{355} \text{ nearly.}$$

In the paper which I sent to the Asiatick Society, and which will appear in the 13th volume of their Researches, the terrestrial arc between Barcelona and Dunkirk, as given in the 2d volume of Colonel MUDGE's Survey, was made use of, and is there stated to be 587987 fathoms, which gives the compression by this method $\frac{1}{272}$. But there must be some mistake in this; for by comparing it with the distance between Montjouy and Dunkirk, as given by DE LAMBRE, the former is considerably greater than the latter, though Montjouy is 3″ south of Barcelona. The mean degree for latitude 47° 24′ used in that paper for determining the ellipticity, compared with the Indian measurements, was deduced from that arc, and gave the compression $\frac{1}{291{,}6}$, while the general mean compression obtained by comparing these measurements with the French, English, and Swedish degrees, was $\frac{1}{304}$ nearly.

Since it is here determined to adopt $\frac{1}{310}$ as the compression, and 60491,46 fathoms for the measure of the degree on the meridian, due to latitude 13° 34′ 44″; we shall have $m' = 60491,46$; $'l = 13° 34' 44''$. Then if m be the degree in any other latitude l; by equation 5, $m = \frac{1 + 3e.\,\text{Sin.}^2\,l}{1 + 3e.\,\text{Sin.}^2\,'l} . m'$. If m have its middle point on the equator, where $l = o$, then

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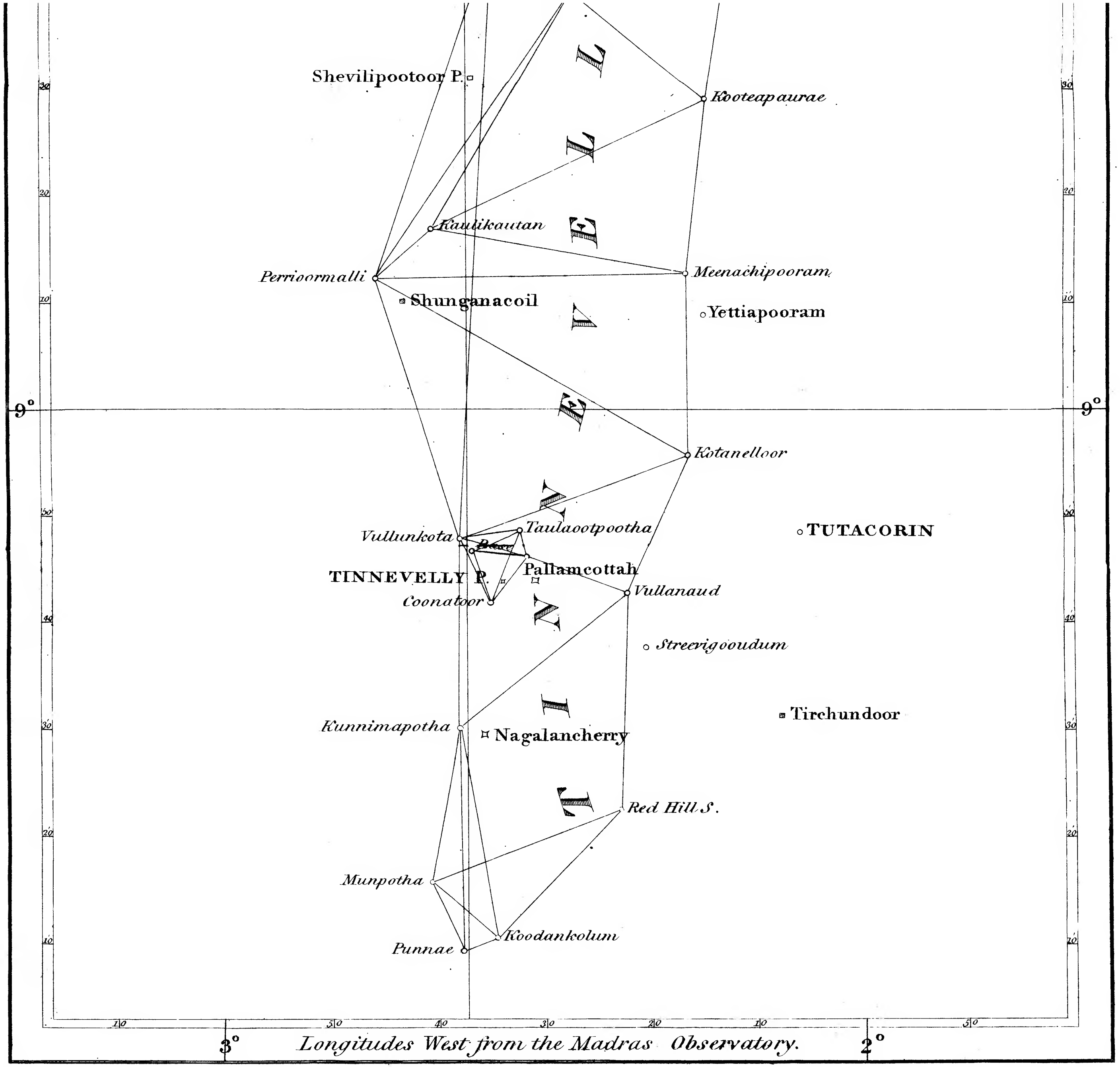
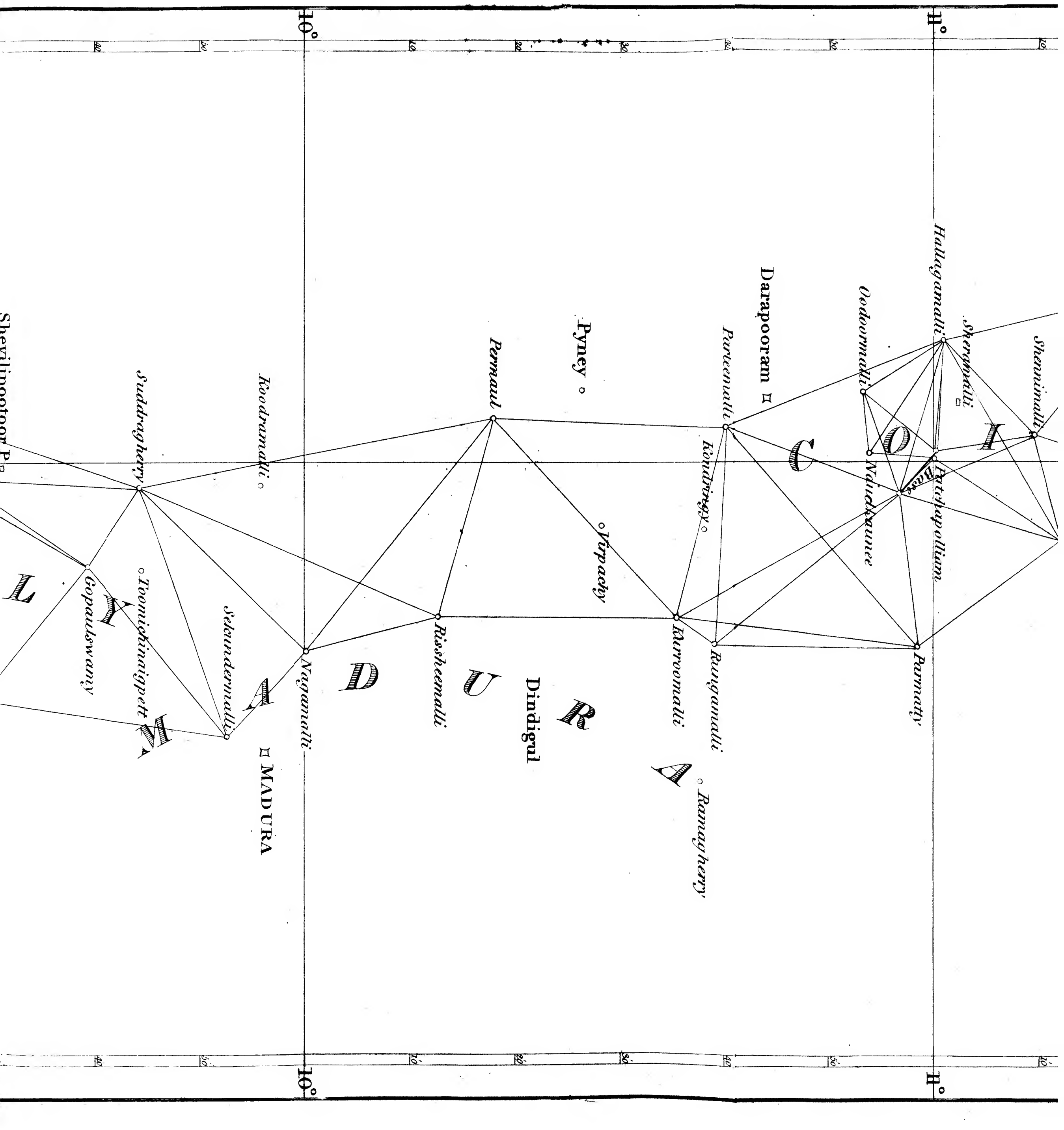
* See vol. iii. p. 89. Base du Systeme Métrique, where the arc between Montjouy and Dunkirk is 551583,6 toises, or 587657,17 fathoms, at the temperature of 32°, which reduced to the temperature of 62°, will be 587475,41 fathoms.

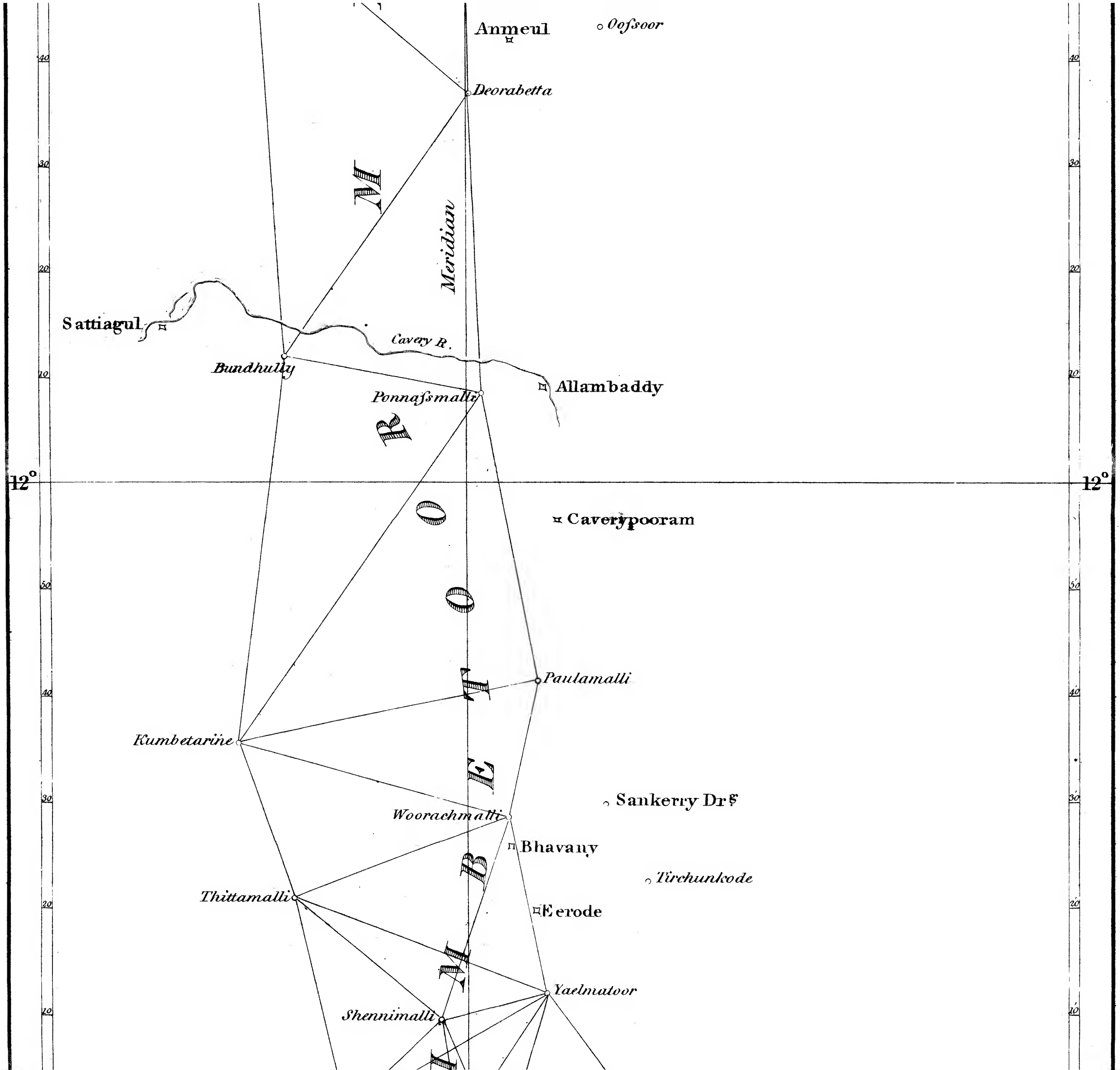
$$m = \frac{m'}{1 + 3e.\,\text{Sin.}^2\,'l} = \frac{60491,46}{1,00053345} = 60459,2 \text{ fathoms.}$$

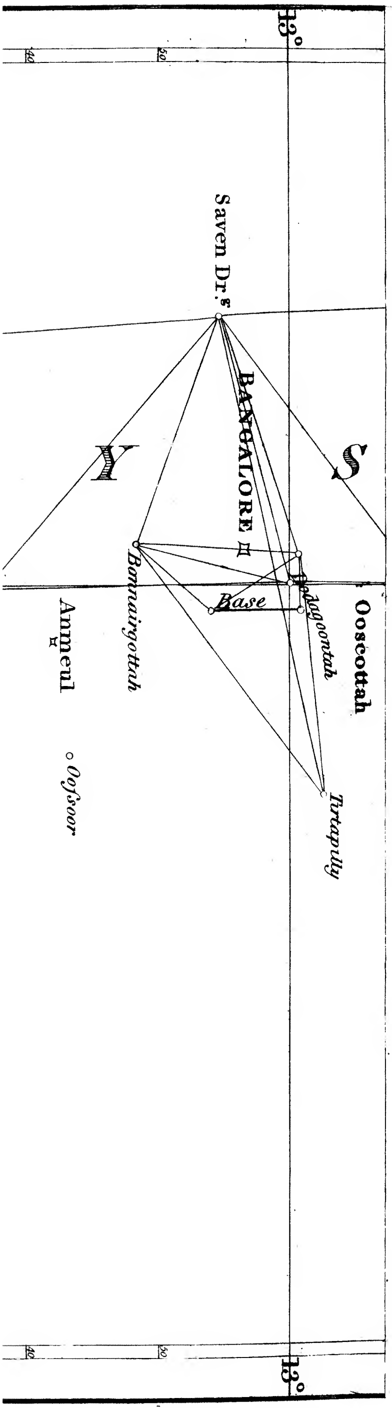
By equation 16, $p = m . \frac{1 + 3e}{1 + e} = 60459,2 . \frac{1 + 3e}{1 + e} =$ $= 60459,2 \times 1,006431 = 60848$ fathoms for the degree on the equatorial circle. Put $A = 57°,2957795$ the arc equal radius. Then $A . p = 57°$, &c. $\times 60848 = 3486334 = \frac{1}{2} a$; and therefore $a = 6972668$ fathoms; and consequently $b\,(= a . (1 - e) = 6972668 \times 9967742 = 6950176$ fathoms, the length of the polar axis. Now since 6972668 is the diameter of the equatorial circle, then 3,14159, &c. multiplied by 6972668, gives 21905280 fathoms for the circumference of the circumscribing the elliptic meridian. Put $d = 1 - \frac{b^2}{a^2} =, 00644$. Then as $1 : 1 - (\frac{d}{2^2} + \frac{3d^2}{2^2 . 4^2}\text{\&c.}) :: 21905280$, the circumference of the circumscribing circle : 21869976 = the circumference of the elliptic meridian; which, divided by 4, gives 5467494 fathoms for the quadrantal arc of that meridian; and this reduced into inches, and divided by 10,000000, will give 39.366 inches for the French métre, at the temperature of 62°. Now, the métre deduced from the measurements of DE LAMBRE and MECHAIN, and reduced from 32° to 62°, was 39,371, English inches, which exceeds this one by ,005 inches: a quantity too small to affect any standard measure: so that the métre as deduced from a comparison of all the recent operations, may be considered, as to practical purposes, the same as that which has been adopted by the French mathematicians, being obtained from comparing the measurements of DE LAMBRE and MECHAIN, with those of BOUGUER and CONDAMINE.

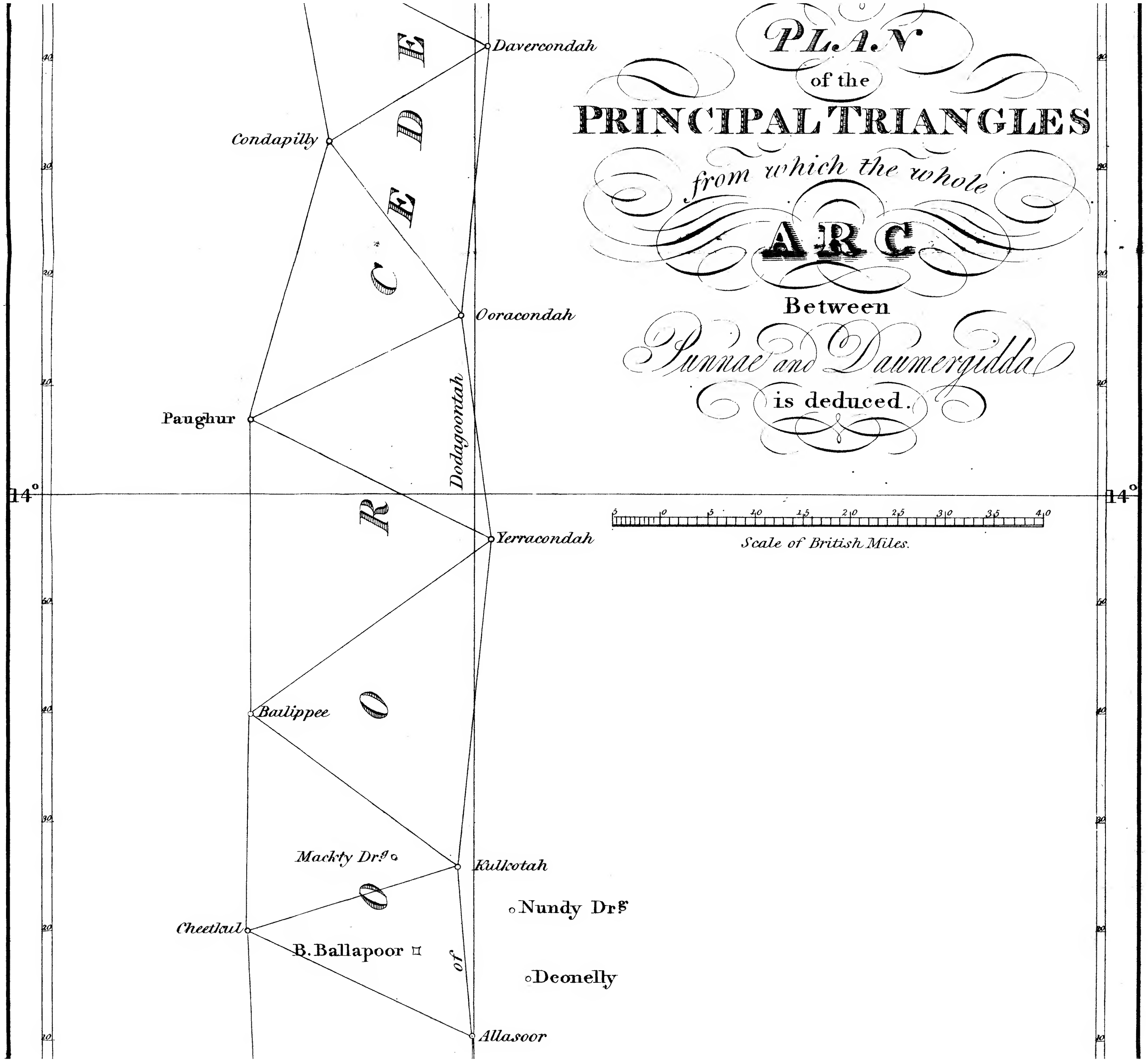
As I am in hopes that another section, and perhaps more, will be added to the arc, I shall defer making any final conclusions till I see what may be done. The next station of observation, I propose to be as near the latitude of 21° 6′ 5″ as possible, in order that the middle point of the section may fall in 19° 34′ 44″, so as to compare the mean degree as obtained by observation, with the one computed from the increments as in the foregoing tables. I say another section, and *perhaps more;* because should the country to the northward be open and settled, there may be a possibility at some future day, of continuing the same arc to the northern confines of Hindostan: so much, at least, seems necessary for laying the foundation of Indian geography; and if it were conducted with zeal and judgment, it would not be a work of many years, provided the features of the country be favourable. The whole time taken up in the measurement of the arc between Punnæ and Daumergidda, including the base lines, astronomical observations, &c.; that is to say, the entire field work, has only been three years and nine months; and a considerable part of the corrections for the stars, for the angles, and for the reduction of the base, were done during the time of measuring the base and observing for the zenith distances; so that I suppose *four years and a half* may be allowed for the whole work. From this estimate, the meridional arc might be continued from Daumergidda to Dhelli in about five years, if no local impediment disturbed its progress. It is however probable that difficulties might occur in Sindia's country, which a northern direction from Daumergidda would render it necessary to pass through. But it would be a sufficient point gained, if a series of triangles were carried from Nagpoor in Berar, to Kalpy on the Jumna, which two places, if the maps are correct, lie nearly on the same meridian. From Kalpy, the meridional series might perhaps be continued north to the Kemaoon mountains; Kalpy would also be a favourable position from which to extend a series to the east and west, and for meridian stations not more than sixty or seventy miles from each other, where the positions of the meridians ought to be determined by pole star observations. Data would then be had for extending the survey on a more enlarged scale over the upper provinces; and the arc between Nagpoor and Kalpy might be easily reduced to the one terminating in latitude 21° 6′ (that station of observation and the one near Nagpoor being connected), so as to form one entire arc between the parallels of Punnæ and Kalpy. Thus would be formed a geometrical connection between the southern and northern possessions of the East India company, and a complete basis laid for local and detailed surveys of the whole. Innumerable might be the individuals employed in carrying them into effect, and various might be the description of these surveys. By the assistance of this work as a foundation, they might all be rendered useful; but without it, no combination of the best common surveys could ever be formed into a correct map to embrace such an extensive territory as British India.


Shevilipootoor P.
Kooteapaurae
Kaulikautan
Perrioormalli
Meenachipooram
Shunganacoil
Yettiapooram
9°
9°
Kotanelloor
TUTACORIN
Vullunkota
Taulaootpootha
Pallamcottah
TINNEVELLY P.
Coonatoor
Vullanaud
Streevigooudum
Tirchundoor
Kunnimapotha
Nagalancherry
Red Hill S.
Munpotha
Koodankolum
Punnae
T I N N E V E L L
10
20
30
40
50
3°
2°
Longitudes West from the Madras Observatory.


10°
11°
40
50
10
20
30
40
50
10
Shenimalli
Sherapmalli
Hallingamalli
Oodoormalli
Darapooram
Purteemalli
Pyney
Permaul
Koodramalli
Suddraghery
Shevilipootoor P
Kondringy
Naduchaune
Pateapollium
Base
Parmatty
Rungamalli
Kurroomalli
Virpachy
Risheemalli
Nagamalli
Sekundermalli
Toomichinaigpett
Gopaulswamy
Ramaghery
Dindigul
MADURA
C O I
M A D U R A
L Y

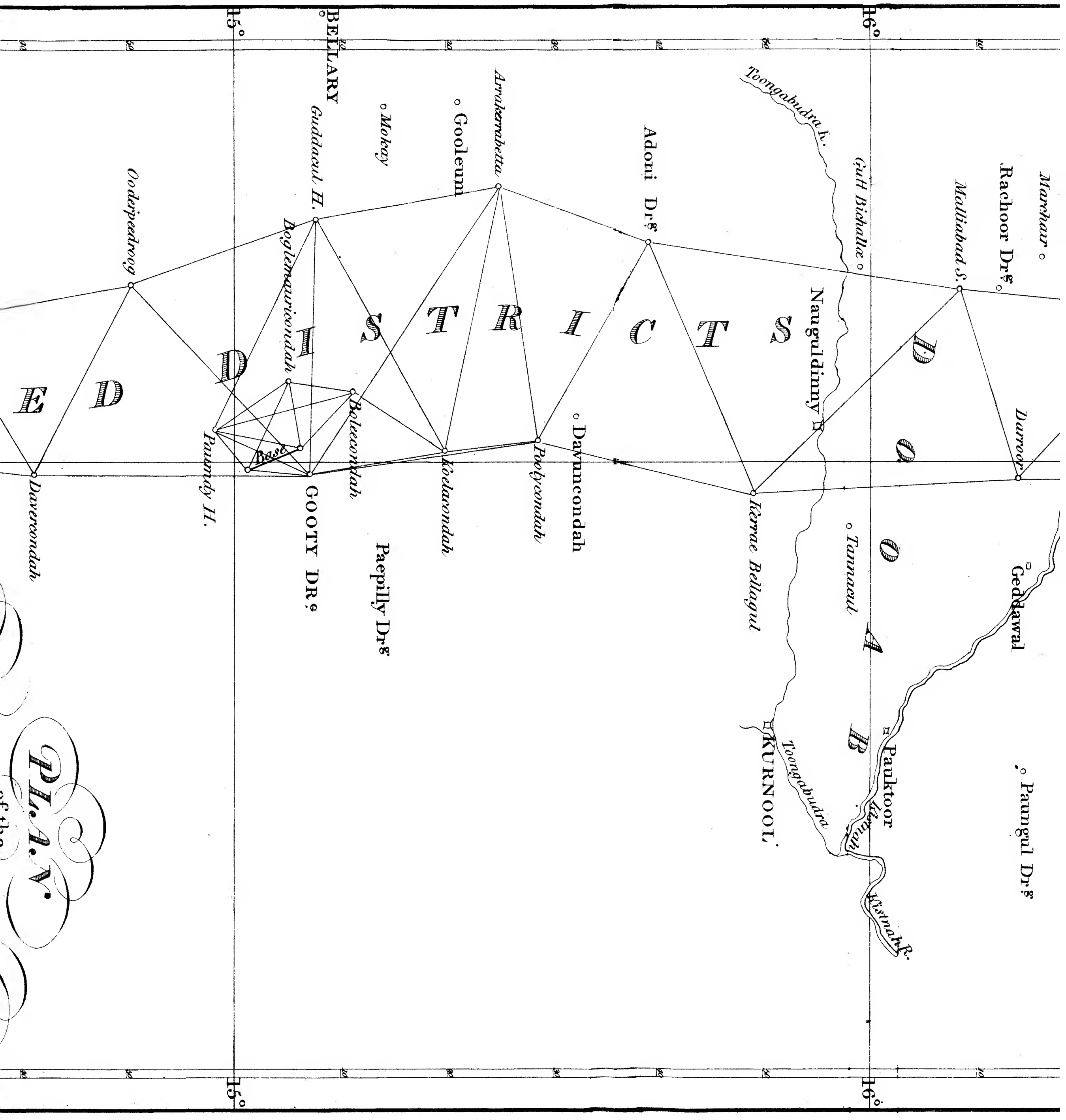

12°
12°
10
20
30
40
50
Anmeul
Oofsoor
Deorabetta
M
Meridian
Sattiagul
Cavery R.
Bundhully
Ponnafsmalli
Allambaddy
R
O
Caverypooram
O
T
Paulamalli
Kumbetarine
E
Sankerry Drs
Woorachmalli
Bhavany
B
Tirchunkode
Thittamalli
Eerode
M
Yaelmatoor
Shennimalli
I

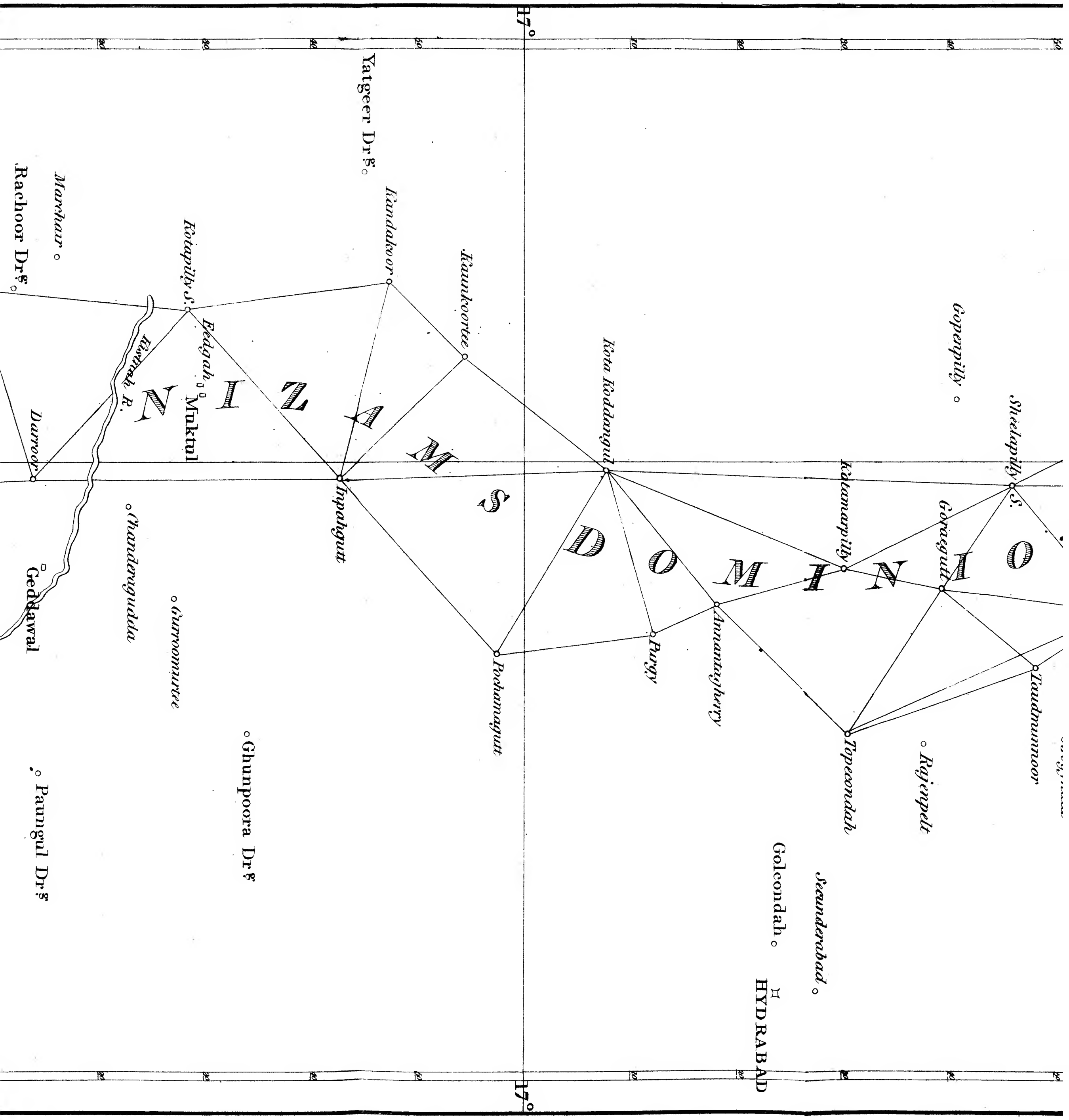

13°
40
50
Saven Dr.g
BANGALORE
Y
S
Bonnairgottah
Base
Jaggoontah
Ooscottah
Anmeul
Ooſsoor
Tirtapully
40
50
13°



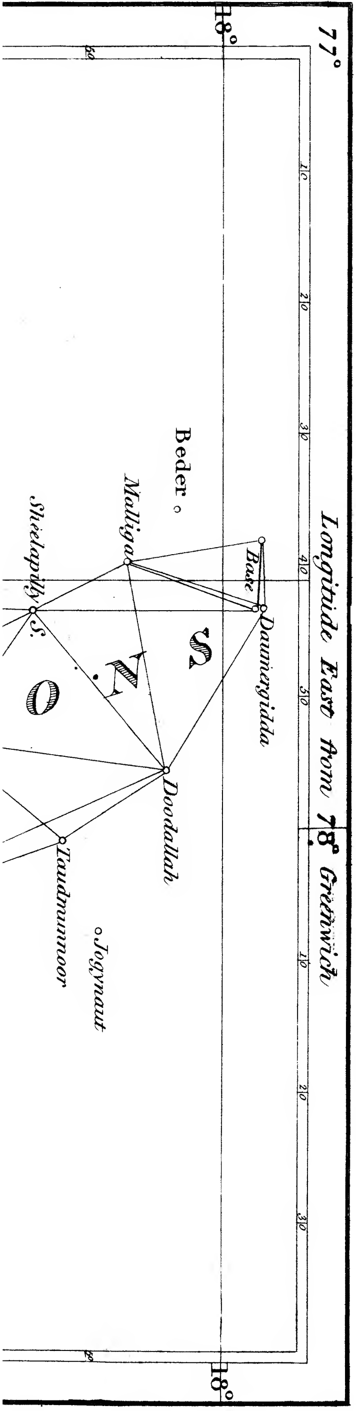
PLAN
of the
PRINCIPAL TRIANGLES
from which the whole
ARC
Between
Punnae and Daumergidda
is deduced.

Scale of British Miles.


15°
16°
10
20
30
40
50
BELLARY
Guddacul H.
Mokey
Gooleum
Arrakerrabetta
Adoni Drg
Toongabudra R.
Gull Bichallie
Rachoor Drg
Marchar
Mulliabad S.
Ooderpeedroog
Boglemauricondah
Nauguldinny
ED DISTRICTS
DOAB
Pasmdy H.
Base
Boleecondah
Koelacondah
Poolycondah
Davuncondah
Davercondah
GOOTY Drg
Paepilly Drg
Kerrae Bellagul
Tunnacul
Darroor
Geddawal
Paungul Drg
Paulktoor
KURNOOL
Toongabudra
Kistnah
Kistnah R.
PLAN
of the


17°
Yatgeer Dr.g
Rachoor Dr.g
Marchaur
Kandakoor
Kotapilly S.
Kaunkoortee
Eedgah
Muktul
Kistnah R.
Darroor
Inpahgutt
Chanderguddah
Gurroomurtee
Gedawal
Ghunpoora Dr.g
Paungul Dr.g
Pochamgutt
Kota Koddangul
Purgy
Annantyhery
Katamapilly
Gopenpilly
Sheelapilly S.
Goraguttt
Taudmunoor
Topecondah
Rajenpett
Golcondah
Secunderabad
HYDRABAD
NIZAMS DOMINIO
10
20
30
40
50

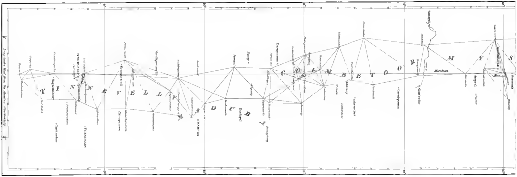
los. Trans. MDCCCXVIII. Plate XXIII. p.516.

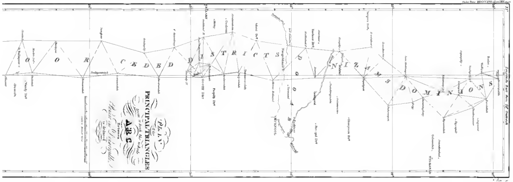


Js. Basire. sc.

The lengths of different degrees computed from the foregoing data, for every three degrees from the equator to the pole.

Lat.	Degrees on the meridian.	Degrees on the perpendicular.	Degrees of Longitude.
0	60459,2	60848,0	60848,0
3	60460,8	60848,4	60765,0
6	60465,6	60850,1	60516,8
9	60473,5	60852,8	60103,6
12	60484,5	60856,5	59526,7
15	60498,4	60861,1	58787,3
18	60515,1	60866,7	57887,7
21	60534,3	60873,2	56830,0
24	60556,0	60880,5	55628,1
27	60579,8	60888,5	54252,0
30	60605,5	60897,1	52738,4
33	60632,7	60906,2	51080,2
36	60661,3	60915,8	49281,9
39	60690,8	60925,7	47348,2
42	60721,3	60935,7	45284,0
45	60751,8	60946,1	43095,4
48	60782,3	60956,4	40787,8
51	60812,5	60966,5	38367,5
54	60842,1	60976,5	35841,1
57	60870,7	60986,1	33215,4
60	60898,0	60995,2	30497,6
63	60923,7	61003,8	27695,2
66	60947,5	61011,8	24815,7
69	60969,1	61018,9	21867,2
72	60988,3	61025,6	18857,9
75	61005,1	61031,0	15796,0
78	61018,9	61035,8	12690,1
81	61029,9	61039,5	9548,7
84	61037,8	61042,1	6380,6
87	61042,6	61043,7	3194,8
90	61044,3	61044,3	—


Longitudes East from the Madras Observatory
T I N N E V E L L Y
TINNEVELLY
Palamcottah
TUTACORIN
M A D U R A
MADURA
Dindigul
C O I M B E T O O R
M Y S O R E
Meridian


Plan of the
PRINCIPAL TRIANGLES
from which the whole
ARC
Between
Punnae and Damargidda
is deduced
O U R   C E D E D   D I S T R I C T S
D O O A B
N I Z A M S   D O M I N I O N S
BELLARY
GOOTY DRY
KURNOOL
HYDERABAD
77°
78°